

ORIGINAL ARTICLE

Sampling method with proportional probability to variable areas in stands of *Eucalyptus dunnii* Maiden in Brazil

Método de amostragem com probabilidade proporcional a áreas variáveis em povoamentos de *Eucalyptus dunnii* Maiden no Brasil

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Abstract

Many forest companies in Brazil started using plots of varying sizes in their continuous forest inventories (CFI), mainly to avoid the effects of marginal trees and to obtain more consistent estimates in the CFI. Consequently it was quite important to present a comparison between the results of applying a sampling method in which trees are selected with probability proportional to a variable average area with the method in which the trees are selected with probability proportional to a fixed area plot. In this study, the theory for a sampling method was developed, in which a selection of a group of trees in a plot is done with probability proportional to a variable average area of occupation (PPVA) per 60 trees in a plot. The data is composed by 41 permanent plots from a forest inventory of *Eucalyptus dunnii* Maiden in Santa Catarina, Brazil. In order to compare the PPVA method with the traditional fixed area method (PPA), the area was imaged with an unmanned aerial vehicle (UAV), allowing to obtain a full list of living trees (census). For this fraction, 7 plots for PPVA method and 7 for the PPA method were also measured, allowing the comparison of the total volume obtained in the census with PPVA and PPA results. The total volume of the plot obtained as a function of the census, when compared to the PPVA method, did not present significant differences at the 99% probability level, while the fixed area method was statistically different at the 95% probability level, resulting in an overestimation of 7.5% higher than that found in the census. The PPAV sampling method provides parameter estimates for characterization of the forest population at a lower cost; it becomes more effective than sampling with fixed area plots, because they are practical and operationally easy for delimitation in the field, providing appropriate estimates and more accurate average occupational area for each individual tree (m²) in the population.

Keywords: PPS sampling; Fixed area plot; Continuous forest inventory.

Resumo

Muitas empresas florestais no Brasil começaram a usar parcelas de tamanhos variados em seus inventários florestais contínuos (CFI), principalmente para evitar os efeitos das árvores marginais e obter estimativas mais consistentes. Consequentemente, foi muito importante apresentar uma comparação entre os resultados da aplicação de um método de amostragem em que as árvores são selecionadas com probabilidade proporcional a uma área média variável com o método em que as árvores são selecionadas com probabilidade proporcional a uma parcela de área fixa. Neste estudo, foi desenvolvida a teoria para um método de amostragem, no qual a seleção de um grupo de árvores em uma parcela é feita com probabilidade proporcional a uma área média variável (PPAV) de ocupação por 60 árvores na

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parcela. Os dados são compostos por 41 parcelas permanentes de um inventário florestal de *Eucalyptus dunnii* Maiden em Santa Catarina, Brasil. Para comparar o método PPVA com o método tradicional de área fixa (PPA), foi fotografada a área com um veículo aéreo não tripulado (VANT), que permitiu a obtenção de uma lista completa de árvores vivas (censo). Para esta fração, também foram medidas 7 parcelas para o método PPVA e 7 para o método PPA, permitindo a comparação do volume total obtido no censo com os resultados do PPVA e PPA. O volume total do talhão obtido em função do censo, quando comparado ao método PPAV não apresentou diferenças significativas ao nível de 99% de probabilidade, ao passo que o método de área fixa apresentou-se diferente estatisticamente ao nível de 95% de probabilidade, resultando em superestimativa de 7,5% maior que o encontrado no censo. O método de amostragem PPAV fornece estimativas de parâmetros para caracterização da população florestal a um custo menor; torna-se mais eficaz do que amostrar com parcelas de área fixa, pois são práticas e operacionalmente fáceis de delimitar em campo, fornecendo estimativas apropriadas e área ocupacional média mais acurada para cada árvore individual (m^2) da população.

Palavras-chave: Amostragem PPS; Parcelas de área fixa; Inventário florestal contínuo.

INTRODUCTION

The continuous forest inventory in companies requires the definition of the method and the sampling process as a structure for obtaining data in a pre-defined area. Regarding the sampling method most used in continuous forest inventories in companies, the use of plots of fixed areas always resulted in border-line-trees or marginal trees, which results in undesirable variations in the estimators obtained per sampling unit. In this way, companies started to use sample units of variable area-sizes in order to avoid such occurrences. However, the forest managers did not consider this problem; that is, they continue to use an average area to obtain the inventory estimates. Based on this evidence, we decided to develop the appropriate methodology to obtain the correct estimators from sample units with probability proportional to average area, called the PPAV method.

We considered as a sampling method the theoretical probabilistic criterion that specifies how trees should be selected to participate in a sampling unit. Bitterlich (1948) proposed the first alternative sampling method to fix-area plot, named thereafter as probability proportional to size (PPS) sampling, in which trees are selected in a circular plot with a probability proportional to basal area, or to $(d_{1,30}^2)$. This sampling method brought new insights for the sampling theory applied to forest inventories and was named, mainly in the United States, as a point sampling. Some other PPS sampling methods followed, varying the size criterion for probability selection of trees, and changing the form of the plot from a circle to a line. The most important contributions are owing to Strand (1958), Cottam & Curtis (1956), and Prodan (1968).

Even though these methods have introduced interesting alternatives to reduce costs of data collection in forest inventories, they present some restrictions, such as bias on the sampling estimates, difficulties in getting dominant heights, and so on. In many forest companies around the world fixed area plots are still the most preferable method for sampling trees in forest inventories to avoid such restrictions.

In some companies in Brazil a variant of the fixed area method (FAM), whose sample units (SUs) are variable in their sizes, is adopted. For example, it will include six or more rows in the planting area with 10 plants per row i.e., approximately 60-hole planting for six rows. In these circumstances, the lines 1 and 6 of the plots will always have 10 trees each, but in lines 2 to 5 would not always have 10 trees each, because the spatial arrangement of trees in each planting row may vary, mainly owing to planting operational conditions and soil relief within the plot. Despite this, the companies have decided, for the sake of consistency of statistical sampling estimates, to completely avoid the occurrence of marginal trees because, in the fixed area sampling method the problem is dealt with in an approximate manner.

The objective of the present study was to provide the theoretical structure of the sampling method, in which individuals in a population are selected with a probability proportional to a variable average area of occupation (PPAV) by 60 trees in the plot and

generate the estimators for density, basal area, and volume. Specifically, we applied this method simultaneously with the FAM to samples of a continuous forest inventory and compared these estimates with those obtained from the census.

MATERIAL AND METHODS

The data used for the present research were collected in an unthinned *Eucalyptus dunnii* stand, in the northern plateau region of Santa Catarina, from a continuous forest inventory carried out in 2015, in an area of 123.9 hectares, located in Três Barras (Latitude: 26° 8'52.94"S and Longitude: 50°20'59.30"W), Santa Catarina, Brazil (Figure 1). The climate in the region of this study is of the Cfb type, according to the Köppen description, with well-distributed rainfall throughout the year and an average temperature of the coldest month below 18°C. Originally, the local vegetation was formed by the Atlantic Forest biome and Mixed Ombrophilous Forest. The relief of the region is slightly undulating, and the soils are predominantly clayey, with good water retention capacity and mostly acidic (Empresa Brasileira de Pesquisa Agropecuária, 2006).

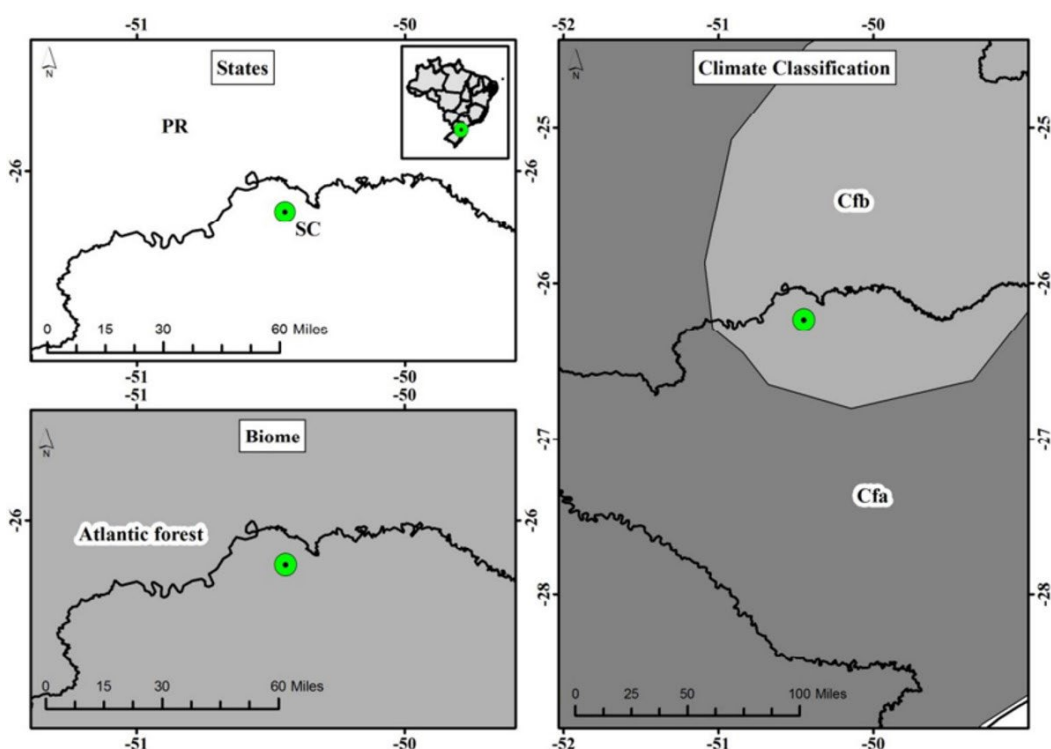


Figure 1: Location, climate and vegetation of the *Eucalyptus dunnii* Maiden stand in Três Barras, SC, Brazil.

The FAM, in which trees are selected with a probability proportional to area (PPA) was compared with the variable area method, in which trees are selected with a probability proportional to the average area (PPAV) of 60 trees per plot.

Data description to elucidate the PPVA sampling method

A total of 41 permanent plots were measured, with dimensions of 6 lines per 10 plants in the planting line, yielding an average variable sample area of 629.01 m² (Table 1). This forest is approximately 6.2 years old, the trees are spaced at 3.0 m × 4.0 m, and the total area is 123.90 hectares.

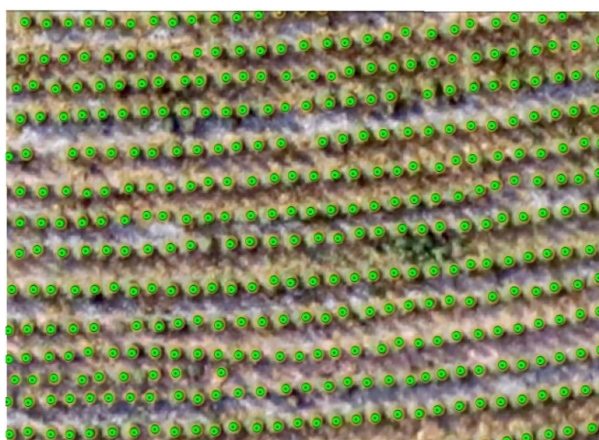
Table 1: Data from compartments (an administrative unit inside the stand delimited by roads and/or streams) of *Eucalyptus dunnii* Maiden in Três Barras, north plateau of Santa Catarina, Brazil

Results	Compartment Number Sampled	Age (years)	Number of Plots	Number of pits	Average Plot Areas per Compartment (m ²)
	01	6.1	2	64	641.23
	02	6.3	2	62	621.27
	03	6.3	4	61	638.20
	04	6.3	2	62	618.11
	05	6.2	1	65	651.33
	06	6.1	6	63	639.00
	07	6.2	1	63	566.17
	08	6.2	3	62	646.03
	09	6.3	3	61	618.03
	10	6.3	2	64	634.52
	11	6.3	3	60	612.66
	12	6.2	5	61	631.70
	13	6.1	3	63	615.86
	14	6.3	1	61	690.80
	15	6.2	3	63	609.25
Total	15	-	41	935	9,434.16
Average		6.2	2.7	62.3	628.94

We used the calculations of mean, variance, standard deviation of the mean, and sampling error based on the complete random sampling design with a maximum admissible error of 10%, at different probability levels of 90, 95, and 99%, whose *t* tabulated values were: 1.684, 2.021, and 2.704, respectively.

Data description for comparison between the variable area and fixed area methods

Images from unmanned aerial vehicle were used to obtain the number of trees per hectare. The images were processed with orthorectification and georeferencing at a very high spatial resolution (25 cm), captured by means of the SenseFly eBee unmanned aerial vehicle (UAV), in *Eucalyptus dunnii* Maiden compartments, adjacent to the study area, aged 6.2 years (Figure 2).

**Figure 2:** Image Detail of High Spectral Resolution and Allocation of Crown Points in a Plantation of *Eucalyptus dunnii* Maiden trees, in Três Barras, Santa Catarina, Brazil.

The total count of individuals (census) was performed manually with the support of the ArcGis 10.2.2 software, where each living tree returned showing on its crown an attribute of item and coordinates of its surface (UTM - Universal Transverse Mercator Coordinates) and allowing to form a specific shapefile with all the trees of the compartment (Figure 3).

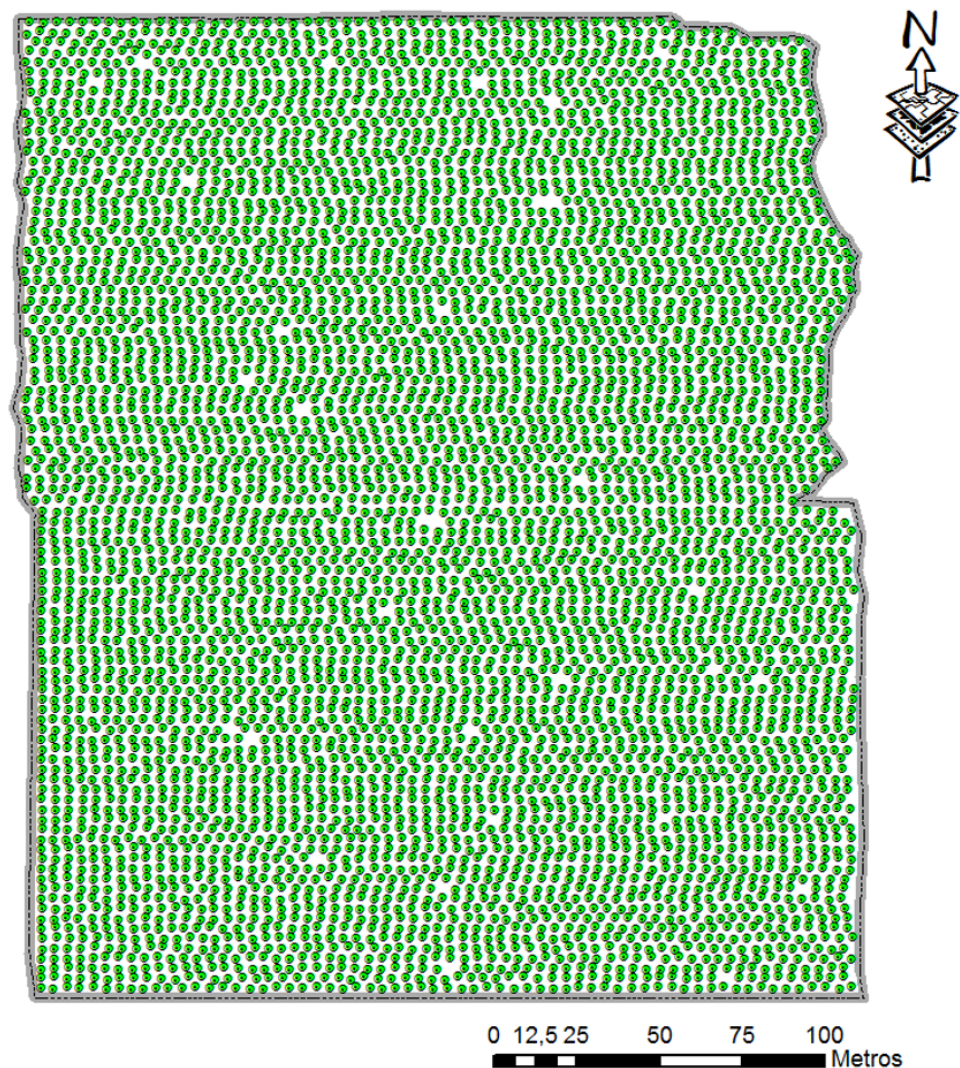


Figure 3: Spatial distribution of 5,952 trees, identified in the image of high spectral resolution from a 7.302-hectare stand of *Eucalyptus dunnii* Maiden, in Três Barras, Santa Catarina, Brazil.

The distribution of trees had an initial spacing of 4.5 m between lines and 3 m between trees in the planting row. However, we detected variations of ~20 cm in these distances, covering, therefore, the limits of 4.30 m < between lines < 4.70 m and 2.80 m < between plants in the row < 3.20 m. Such variations, observed in a reformation area of second rotation, are owing to steepness, regulation of the planting machines, planting mechanization activity, and the route deviations between the rows.

Subsequently, seven plots with a total area of 700 m² were randomly allocated in the stand. Other seven plots with a variable area, composed by 6 planting lines with 10 plants in each line, were allocated randomly (Figure 4A and Figure 4B), leading to a density of approximately one plot per hectare. The total number of trees within plots were counted in the same way as in the census previously described.

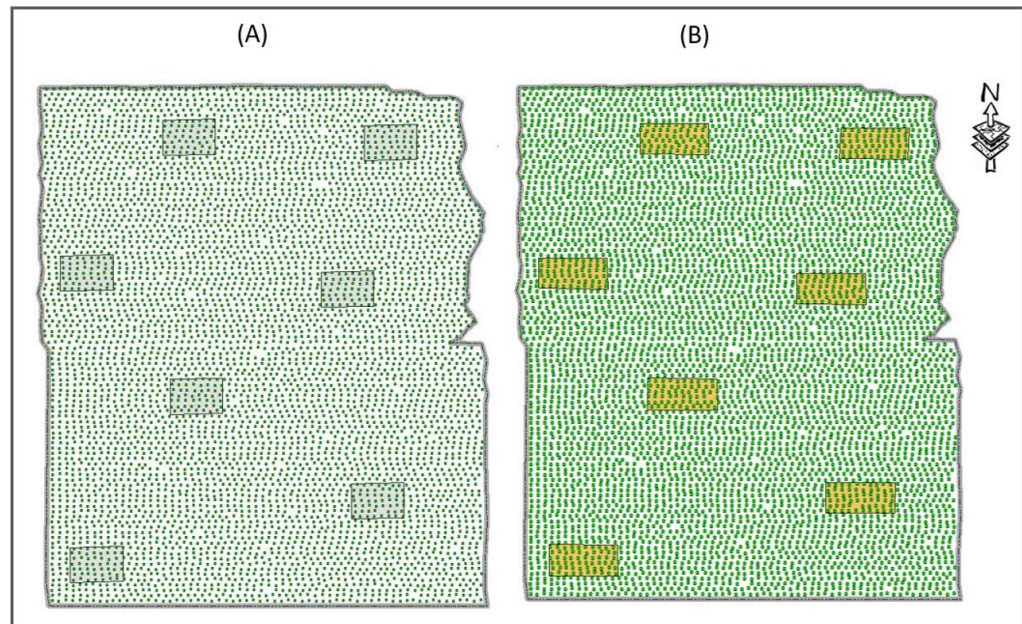


Figure 4: Random distribution of 7 plots of fixed area (A) and 7 plots of variable area (B) in a stand of *Eucalyptus dunnii* Maiden, in Três Barras, Santa Catarina, Brazil. Where: A is the distribution of the sampled fixed area plots in the compartment; B is the distribution of the sampled variable area plots in the compartment.

Theoretical development of the variable area method

This proposal comes after many reflections and evaluations on the *pps* sampling methods already known in the literature, as well as on the *FAM*. In many practical applications in forest inventories, it was detected that *FAM* presents undesirable practical circumstances, as mentioned before, mainly the occurrence of marginal trees and incorrect evaluation of the average coverage area (m²) of the trees in the population.

Probability criteria for tree selection

The probability criterion for tree selection is proportional to its average occupation area in the plot. To ensure equal probability of selection of trees in the stand, we proceeded as follows:

- a) Each plot was randomly selected, such that its left side was identified as one planting line, starting with one tree, and following up to the next ten subsequent trees. In the same way, another ten trees were taken in the sixth line selected on the right side of the previous one. Subsequently, the plot was closed on the limits of the intermediate lines, that is, from the 2nd to the 5th, in such a way that there were no marginal trees,
- b) The distances between and along the lines resulting from the final fit were measured and formed a rectangle; the product of its side lengths resulted in the variable area of the plot, i.e., A_i ,
- c) Once the limits of the plot were closed, the trees in the 2nd to 5th lines were counted to obtain the variable number of individuals in the plot, that is, n_i . If A_i had C_i established planting holes, then $n_i \leq C_i$,
- d) As can be noted, in each plot the selection of trees was conditioned to an average area a_i , associated to the tree condition X_i , such that we could establish the probability of its occurrence, which was obtained by relations (1) and (2).

$$a_i = \frac{A_i}{n_i} \quad (1)$$

$$p_i = \frac{a_i}{A} \quad (2)$$

Where A is the area of the sampled population.

If A is one ha, then the inverse of the probability defined in (2) would be the estimator of the number of trees per hectare in each plot, i.e.,

$$\frac{1}{p_i} = p^{-1} = \frac{A}{a_i} = \frac{An_i}{A_i} = N_i \quad (3)$$

It should be noted that this estimator is variable for each plot, typically a method of sampling with probability proportional to size (pps), which in this case is the area a_i .

In this sampling procedure, in area A , a dichotomous condition occurs, that is, the plot is represented by the average area a_i , to which the trees that integrate it occur with equal probability and such a set will be considered equal to one or success and the other sets of trees ($M-1$) in the forest that do not qualify for the sampling will be considered equal to zero, or failure. Under these circumstances, we can obtain the mathematical expectation for the total number of trees per hectare N , or $E(\sum_{i=1}^M X_i)$:

$$E\left(\sum_{i=1}^M X_i\right) = \sum_{i=1}^{M-1} [(X_i = 0)p^{-1}] + \sum_{i=1}^1 [(X_i = 1)p^{-1}] = \sum_{i=1}^1 \frac{An_i}{A_i} = \frac{10,000 n_i}{A_i}$$

Therefore:

$$N_i = 10,000 n_i A_i^{-1} \quad (4)$$

It is considered that, to obtain the estimators of basal area and volume per hectare, mathematical expectations are obtained in a similar manner.

The mathematical expectation of the basal area will be obtained by summing all possible sets of trees, from which their average transversal areas are taken, i.e.

$$(\sum_{i=1}^{n_i} g_i) n_i^{-1} = \bar{g}_i \quad (5)$$

If this is taken in the total forest area $E(\sum_{i=1}^M \bar{g}_i X_i)$ will be obtained as follows:

$$\begin{aligned} E\left(\sum_{i=1}^M \bar{g}_i X_i\right) &= \sum_{i=1}^{M-1} [\bar{g}_i (X_i = 0)p^{-1}] + \sum_{i=1}^1 [\bar{g}_i (X_i = 1)p^{-1}] \\ E\left(\sum_{i=1}^M \bar{g}_i X_i\right) &= \sum_{i=1}^1 \bar{g}_i p^{-1} = \bar{g}_i p^{-1} = \bar{g}_i \frac{An_i}{A_i} = \frac{10,000 \bar{g}_i n_i}{A_i} \\ \hat{G}_i &= 10,000 \bar{g}_i n_i A_i^{-1} \end{aligned} \quad (6)$$

The mathematical expectation of the volume will be obtained by the sum of all possible combinations of n_i trees, from which we take their individual volumes, i.e.

$$(\sum_{i=1}^{n_i} v_i) n_i^{-1} = \bar{v}_i \quad (7)$$

The mathematical expectation of the volume $E(\sum_{i=1}^M \bar{v}_i X_i)$ will be obtained in (8):

$$E\left(\sum_{i=1}^M \bar{v}_i X_i\right) = \sum_{i=1}^{M-1} [\bar{v}_i (X_i = 0) p^{-1}] + \sum_{i=1}^1 [\bar{v}_i (X_i = 1) p^{-1}]$$

$$E\left(\sum_{i=1}^M \bar{v}_i X_i\right) = \sum_{i=1}^1 \bar{v}_i p^{-1} = \bar{v}_i p^{-1} = \frac{A \bar{v}_i n_i}{A_i} = \frac{10,000 \bar{v}_i n_i}{A_i}$$

$$V_i = 10.000 \bar{v}_i n_i A_i^{-1} \quad (8)$$

RESULTS AND DISCUSSION

Variable area method

The sampling method PPVA was applied to data from 41 plots and the results are presented in Table 2. The number of pits varied from 58 to 65, with an average of 62.49. The number of trees per plot varied from 30 to 62, with average 56.66.

The sum of the transversal areas of the trees per plot varied from 0.687 m² to 1.623 m², with average 1.312 m². The volume of the trees per plot varied between 8.000 m³ to 21.196 m³, with average of 15.597 m³ (Table 2).

Table 2 shows that even when dealing with a stand from the same annual production unit, silvicultural techniques, and initial spacing (3.5 m × 3.0 m), a high variability of the plot areas occurred, ranging from 555.84 to 690.80 m², with average of 629.00 m². It was also evident that there was a variation of the initial density of planting, even allowing to present inferences about the silvicultural consistency at the time of its implementation, a characteristic that would not be explicitly evident in the FAM.

The estimators of PPVA sampling are presented in Table 3. The sample estimators resulted in an average density of 902 trees·ha⁻¹, average basal area of 20.88 m²·ha⁻¹, and average volume of 248.167 m³·ha⁻¹.

Table 2: Results of the estimators obtained from 41 plots of *Eucalyptus dunnii* Maiden in Tres Barras, Santa Catarina, Brazil.

Stand Number	Plot Number	Pits in the Plots	Trees per Plots n_i	Plot area A_i (m ²)	Average area per tree $A_i = A_i/n_i$ (m ²)	Sum of the transversal areas (m ²)	Sum of the trees volumes (m ³)
3	1	61	59	658.5	11.2	1.505	19.312
3	2	62	57	584.0	10.2	1.272	13.929
8	3	60	60	634.8	10.6	1.552	21.044
8	4	60	60	651.2	10.9	1.548	21.196
8	5	61	59	607.4	10.3	1.261	16.596
10	6	65	60	637.7	10.6	1.415	17.062
10	7	58	57	598.5	10.5	1.244	15.306
2	8	65	49	598.5	12.2	1.167	13.422
8	9	62	60	659.4	11.0	1.349	15.693
2	10	62	60	684.0	11.4	1.242	12.059
19	11	58	53	597.2	11.3	1.347	15.451
11	12	65	56	651.3	11.6	1.262	13.276
19	13	63	55	621.1	11.3	1.513	20.524
19	14	63	52	635.8	12.2	1.244	14.652

Table 2: Continued...

Stand Number	Plot Number	Pits in the Plots	Trees per Plots n_i	Plot area A_i (m ²)	Average area per tree $A_i = A_i/n_i$ (m ²)	Sum of the transversal areas (m ²)	Sum of the trees volumes (m ³)
12	15	64	59	670.2	11.4	1.266	14.342
12	16	62	62	648.8	10.5	1.623	20.404
12	17	64	61	650.5	10.7	1.151	12.076
18	18	60	58	640.9	11.0	1.321	14.225
12	19	62	61	613.8	10.1	1.493	18.387
12	20	60	49	602.0	12.3	1.109	11.125
12	21	64	52	648.8	12.5	1.182	12.826
18	22	61	56	630.7	11.3	1.304	13.881
18	23	64	57	666.6	11.7	1.269	12.696
34	24	63	62	602.0	9.7	1.462	19.009
34	25	63	43	623.2	14.5	1.077	13.622
34	26	64	60	602.6	10.0	1.584	20.742
15	27	63	61	566.2	9.3	1.243	14.807
33	28	61	53	690.8	13.0	1.476	18.785
31	29	61	60	591.5	9.9	1.345	16.603
32	30	61	55	632.3	11.5	1.316	17.417
32	31	63	56	611.1	10.9	1.176	12.866
21	32	64	53	639.9	12.1	1.388	18.941
31	33	62	61	612.9	10.0	1.236	15.065
31	34	62	60	648.2	10.8	1.478	18.341
32	35	64	30	604.2	20.1	0.687	8.000
21	36	64	60	629.1	10.5	1.480	19.866
28	37	60	59	678.2	11.5	1.150	11.101
31	38	59	57	633.4	11.1	1.371	15.958
31	39	63	62	672.4	10.8	1.446	16.790
28	40	60	58	555.8	9.6	1.105	11.244
28	41	61	61	603.9	9.9	1.146	10.845

Table 3: Results of the application of the PPAV method in 41 plots of *Eucalyptus dunnii* Maiden in Tres Barras, Santa Catarina, Brazil, for average of transversal area ($\bar{g}_i$), average of volume ($\bar{v}_i$), number of the trees per hectare (N_i), basal area (G_i) and volume per hectare (V_i).

Stand N°	Plot N°	$\bar{g}_i$ (m ²)	$\bar{v}_i$ (m ³)	N_i (ha)	G_i (m ²)	V_i (ha)
3	1	0.0255	0.3273	896	22.85	293.266
3	2	0.0223	0.2444	976	21.78	238.509
3	3	0.0259	0.3507	945	24.45	331.521
8	4	0.0258	0.3533	921	23.76	325.482
8	5	0.0214	0.2813	971	20.75	273.220
8	6	0.0236	0.2844	941	22.20	267.552
10	7	0.0218	0.2685	952	20.79	255.738
10	8	0.0238	0.2739	819	19.50	224.269
10	9	0.0225	0.2616	910	20.46	237.999
2	10	0.0207	0.2010	877	18.16	176.316
8	11	0.0254	0.2915	887	22.56	258.708
2	12	0.0225	0.2371	860	19.37	203.833
19	13	0.0275	0.3732	886	24.36	330.460
11	14	0.0239	0.2818	818	19.57	230.443

Table 3: Continued...

Stand N°	Plot N°	$\bar{g}_i$ (m ²)	$\bar{v}_i$ (m ³)	N_i (ha)	G_i (m ²)	V_i (ha)
19	15	0.0215	0.2431	880	18.88	214.001
19	16	0.0262	0.3291	956	25.01	314.508
12	17	0.0189	0.1980	938	17.70	185.661
12	18	0.0228	0.2453	905	20.62	221.960
12	19	0.0245	0.3014	994	24.33	299.549
18	20	0.0226	0.2270	814	18.42	184.796
12	21	0.0227	0.2466	802	18.22	197.692
12	22	0.0233	0.2479	888	20.68	220.110
12	23	0.0223	0.2227	855	19.03	190.465
18	24	0.0236	0.3066	1030	24.28	315.765
18	25	0.0250	0.3168	690	17.28	218.579
34	26	0.0264	0.3457	996	26.29	344.234
34	27	0.0204	0.2427	1077	21.96	261.532
15	28	0.0278	0.3544	767	21.36	271.929
15	29	0.0224	0.2767	1014	22.73	280.701
34	30	0.0239	0.3167	870	20.81	275.449
15	31	0.0210	0.2297	916	19.24	210.537
33	32	0.0262	0.3574	828	21.69	296.001
31	33	0.0203	0.2470	995	20.16	245.799
32	34	0.0246	0.3057	926	22.80	282.939
32	35	0.0229	0.2667	497	11.36	132.411
21	36	0.0247	0.3311	954	23.53	315.768
21	37	0.0195	0.1881	870	16.96	163.667
31	38	0.0241	0.2800	900	21.65	251.928
31	39	0.0233	0.2708	922	21.50	249.706
32	40	0.0191	0.1939	1043	19.89	202.284
21	41	0.0188	0.1778	1010	18.97	179.578
Total	-	-	-	36,996	855.93	10,174.864
Average		0.0232	0.2756	902	20.88	248.167

The variations in the number of pits, number of plants and area of plots observed using the method with variable areas explain why the results of the sampling are closer to the census values, that is, with greater accuracy.

The statistical analysis by complete random sampling procedure, applied to 41 plots is presented in Table 4, and the results show that this sample size produced a standard error of less than the maximum admissible error value of 10%, whose values were 5.48, 6.57, and 8.80%, respectively at probability levels of 90, 95 and 99%, respectively.

Table 4: Statistical analysis of the results with the application of PPVA method in 41 plots of *Eucalyptus dunnii* Maiden in Três Barras, Santa Catarina, Brazil.

Population	Area (ha)	Volume (m ³ ·ha ⁻¹)	Variance (m ³ ·ha ⁻¹) ²	Standard Deviation (m ³ ·ha ⁻¹)	Standard Error (m ³ ·ha ⁻¹)	Standard Error %
Mean	123.90	248.167	2,716.764	52.123	8.140	-
Minimum	-	132.411	-	-	-	-
Maximum	-	344.234	-	-	-	-
Probability 90%	-	-	-	-	13.593	5.48%
Probability 95%	-	-	-	-	16.315	6.57%
Probability 99%	-	-	-	-	21.832	8.80%

The confidence intervals for the volume in m³·ha⁻¹ and for the total population of 123.90 ha, are presented in Table 5, from where it is also possible to verify the *t* values of the Student distribution and the sample sizes required to meet each probability level.

Table 5: Confidence intervals for the volume estimators obtained with PPVA sampling method in 41 plots of *Eucalyptus dunnii* Maiden in Três Barras, Santa Catarina, Brazil.

Estimator	t values	Required Sample Size	Confidence Intervals
Mean (90%)	1.684	12	$234.57 \leq X \leq 261.76$
Mean (95%)	2.021	18	$231.85 \leq X \leq 264.48$
Mean (99%)	2.704	32	$226.34 \leq X \leq 270.00$
Total (90%)	-	-	$29,063.77 \leq X \leq 32,432.76$
Total (95%)	-	-	$28,726.48 \leq X \leq 32,769.40$
Total (99%)	-	-	$28,042.97 \leq X \leq 33,452.92$

Comparison of the fixed area and variable area methods with the results of the census

A total of 5,952 live trees were identified and counted in the census with the high-resolution image. Also, the delimitation of the compartment polygon allowed to accurately extract the net area of planting, as well as the density of trees per hectare and for the total area sampled (Table 6). In addition, with the average volume (m³) of the trees sampled in the plot, it was possible to estimate the volume per hectare and for the total area.

Table 6: Results of the census conducted in a stand of *Eucalyptus dunnii* Maiden in Três Barras, Santa Catarina, Brazil

N° Trees (ha)	Stand area (ha)	N° Trees (total)	Volume (m ³ per tree)	Volume (m ³ .ha ⁻¹)	Total Volume (m ³)
819.5	7.302	5,984	0.325	266.3	1,944.8

With the simulation of fixed area plots, the results shown in Table 7 were obtained, in which a total of 6,433 trees was estimated by the method, showing a total volume for the compartment of 2,090.600 m³.

Table 7: Density and volume obtained with the FAP in a stand of *Eucalyptus dunnii* Maiden in Três Barras, Santa Catarina, Brazil

Pits per Plot	Plot Area (m ²)	N° of Trees (ha)	Stand Area (ha)	N° of Trees (total)	Volume per ha (m ³ ha ⁻¹)	Total Volume (m ³)
61	700	866.7	7.302	6,328	281.700	2,056.700
63	700	900.0	7.302	6,572	292.500	2,135.800
64	700	916.7	7.302	6,694	297.900	2,175.400
61	700	883.3	7.302	6,450	287.100	2,096.300
63	700	900.0	7.302	6,572	292.500	2,135.800
62	700	883.3	7.302	6,450	287.100	2,096.300
57	700	816.7	7.302	5,963	265.400	1,938.100
Average	700	881.0	7.302	6,433	286.300	2,090.600

The simulation performed based on the variable area plot (*PPAV*) is presented in Table 8, in which a total of 6,034 trees and a volume of 1,961.100 m³ were estimated.

Table 8: Density and volume obtained with *PPAV* in a stand of *Eucalyptus dunnii* Maiden in Três Barras, Santa Catarina, Brazil.

Pits per Plot	Plot Area (m ²)	Nº Trees (ha)	Plot Area (ha)	Nº Trees (total)	Volume per ha m ³ ha ⁻¹	Total Volume (m ³)
60	726.2	826.2	7.302	6,033	268.500	1,960.700
59	739.3	798.1	7.302	5,827	259.400	1,893.900
61	727.4	838.6	7.302	6,123	272.500	1,990.100
60	723.7	829.1	7.302	6,054	269.400	1,967.500
59	711.1	829.7	7.302	6,058	269.700	1,969.000
59	720.2	819.2	7.302	5,982	266.200	1,944.100
60	711.1	843.8	7.302	6,161	274.200	2,002.400
Total	722.7	826.4	7.302	6,034	268.600	1,961.100

Comparative results of the analysis of variance between the census, *FAM*, and *PPAV* methods are presented in Table 9.

Table 9: Density and volume obtained in variable area plots in a stand of *Eucalyptus dunnii* Maiden in Três Barras, Santa Catarina.

Method	Nº of Trees (ha)	Compartment Area (ha)	Nº of Trees (total)	Volume per ha (m ³ .ha ⁻¹)	Total Volume (m ³)	Difference (%)
Census	819.5		5,984	266.3	1,944.800	-
<i>FAM</i>	879.6	7.302	6,423**	285.9**	2,087,400**	7.3%
<i>PPVA</i>	826.4		6,034 ^{ns}	268.6 ^{ns}	1,961,100 ^{ns}	0.8%

* Significant at 95%; ** significant at 99%; ^{ns} not significant.

The estimates were highly accurate when the variable area plot (*PPVA*) was used to obtain the density and total volume of the evaluated compartment, with no significant differences at 95 and 99% probability levels. The *FAM* presents bias, overestimating in 7.3% the density and total volume of the evaluated compartment and presenting an outcome significantly different from that obtained in the census, at 99% probability level.

Sampling method: Probability proportional to a variable area (*PPAV*)

Although the literature on Brazilian forestry does not specify or use the terminology of sampling with probability proportional to a variable area (*PPAV*), as a new sampling method, with estimators appropriately developed by mathematical expectation, it is well known that variable plot size has been used in practice for some time. Mello et al. (2009) applied two sampling methodologies: random sampling and geostatistics to evaluate the sampling behavior with fixed area plots and in-line plots (120 m by 2 planting lines, thus variable area) to obtain the volume in m³ of approximately 104 ha of *Eucalyptus grandis* plantation in the state of São Paulo, Brazil. The conversion of the estimates per hectare was made by taking the average area of the plots and finding, afterwards, the average conversion factor as is done for the *FAM*.

Londero et al. (2015) used data from plots with variable area to calibrate the 3-PG model for *Eucalyptus saligna* Smith in the region of Guaíba-RS, Brazil, composed of 6 lines with n_i trees, totaling 60 holes planted evaluated in each plot in their study. They used variable area plots, but they have not used a *pps* sampling structure to estimate any quantitative variables for the forest stands, therefore they did not formulate a conversion area factor to obtain estimates per hectare.

Almeida et al. (2016) tested sampling with fixed area plots and with plots whose areas were defined by the fixed number of hole planting in a stand of eucalyptus in MG, Brazil. They tested the values obtained in these two plot sizes and found no statistically significant differences between them and concluded that these two sampling methods would result in equally consistent estimates per hectare. Therefore, they applied the conversion factor normally used for fixed area plots to obtain the estimates per hectare.

Moscovich et al. (1992) referred to Bitterlich, Strand, Prodan, and Quadrant sampling methods framing them as variable area methods in a comparative study carried out in an *Araucaria angustifolia* forest in Rio Grande do Sul, Brazil. In the international literature it is also common to find the mention of variable area when referring to Bitterlich and Prodan sampling methods (Husch et al., 2003; Hamilton & Bensted-Smith, 1989). In our research the concept of variable area plot refers to a consequent resulting plot size after selecting a fixed number of trees in the forest stand.

Batcheler & Craib (1985) tested the Prodan's variable area and *FAM* with different plot sizes to obtain estimates of the number of trees per hectare and the basal area with data from native forests of New Zealand, and concluded that variable and small area sampling is robust and suitable for ecological research of populations used in their studies. Once more, the concept of variable area in this study refers to Prodan's sampling method, certainly a *pps* sampling, in which the selection of trees to participate in the plot is applied to a set of trees, usually 6 trees, in which the probability is proportional to the radius of the 6th tree in a circle randomly selected in the stand.

Alijanpour et al. (2018) compared the performance of the *FAM* and the Bitterlich variable area method to assess quantitative and qualitative biodiversity in a sub-humid forest in northwestern Iran. The results indicated that the quantitative characteristics are significantly different between the two methods, whereas there were no significant differences in richness, biodiversity indices and qualitative characteristics. Again, the concept of variable area in this research refers to Bitterlich's sampling method, certainly a *pps* sampling, in which the selection of trees to participate in the plot is done with a probability proportional to the transversal area of each tree, or (d^2) in a circle randomly selected in the stand. Similarly, Fallah et al. (2000) compared two sampling methods with fixed area (10 acres) and variable area (Bitterlich) plots in oak forests in Persia, assessing the number of trees per hectare, basal area and time required to record data. The results showed that the 10-acre sampling method was more adequate for the forest inventory.

The most important influence on the estimates is the average occupation area (m^2) of the trees in the population, while the sampling with variable area can identify variations of spacing specific to each plot and, therefore, generate more consistent estimates to the compartment.

The variable area sampling method is already a reality in several forest companies in Brazil, among them the Eco-Brazil Forests in Tocantins, Investment groups in Maranhão, and F. Plywood factory S.A in Tres Barras, Santa Catarina, however they are still considering the estimates as if they were taken in fixed area plots.

Given the possible variability of existing gaps in the forestry companies, whether they are caused by slope, soil types, equipment, and other silvicultural practices that exert influence on tree coverage average area in the population, it is recommended to use the variable area sampling method to generate more efficient and accurate estimators in the referred areas.

The methodology with selection of trees proportional to a variable area (PPAV) could significantly mitigate the problem of marginal trees in the plots, correct possible systematic errors of spacing, and operationally facilitate the implementation and re-measurement of plots. As the variable area plots do not contain marginal trees, they provide better conditions

to obtain the CFI estimates, such as representativeness and stability of diametric, hypsometric, and volumetric distributions, as well as mortality and in-growth control, among others. The sample structure assures uniformity, or fixation of the same plot in different types of spacing, guaranteeing the estimates to maintain fairness of results despite the variation in spacing. This sample structure guarantees minimum representation of trees in the plot to generate their estimates with precision and statistical consistency. The most influential feature of PPAV sampling is the average occupancy area (m²) of trees in the population, which can identify spatial variations in each plot and thus generates more consistent estimates to the stand.

CONCLUSIONS

PPAV sampling method provides the best parameter estimates for characterization of the forest population using variable area plots.

PPAV is more effective than sampling with fixed area plots, because its plot structure is easier to instal in the field.

Using an average area of variable-size plots as if fixed-size plots have being used in the sampling is inappropriate.

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